

# The small mixing angle $\theta_{13}$ and the lepton asymmetry

Song-Haeng Lee\* and Kim Siyeon†

*Department of Physics, Chung-Ang University, Seoul 156-756, Korea*

(Dated: May 1, 2005)

## Abstract

We present the correlation of low energy CP phases, both Dirac and Majorana, and the lepton asymmetry for the baryon asymmetry in the universe, with a certain class of Yukawa matrices that consist of two right-handed neutrinos whose mass ratio is about  $2 \times 10^{-4}$  and include one texture zero in themselves. For cases in which the amount of the lepton asymmetry  $Y_L$  turns out to be proportional to  $\theta_{13}^2$ , we consider the constraint between two types of CP phases and the relation of  $Y_L$  versus the Jarlskog invariant or the amplitude of neutrinoless double beta decay as  $\theta_{13}$  varies.

PACS numbers: 11.30.Fs, 14.60.Pq, 14.60.St

---

\*Electronic address: shlee@phys.cau.ac.kr

†Electronic address: siyeon@cau.ac.kr

## I. INTRODUCTION

A study of neutrino masses is launched from the result which is that two of three mixing angles in the lepton sector are large and the other is small. Interpreting the atmospheric and solar neutrino experiments [1][2] in terms of two-flavor mixing, the mixing angle for the oscillation of atmospheric neutrinos is understood to be maximal or nearly maximal:  $\sin^2 2\theta_{\text{atm}} \simeq 1$ , whereas the one for the oscillation of solar neutrinos is not maximal but large:  $\sin^2 \theta_{\text{sol}} \simeq 0.3$ . The masses of charged leptons are the most precisely measured parameters of the fermions. The data reads  $m_e = 0.51$  MeV,  $m_\mu = 106$  MeV,  $m_\tau = 1780$  MeV. Meanwhile, as for neutrinos, the existing data just show that the neutrino mass squared differences which induce the solar and atmospheric neutrino oscillations are  $\Delta m_{\text{sol}}^2 \simeq (7_{-2}^{+10}) \times 10^{-5}$  eV<sup>2</sup> and  $\Delta m_{\text{atm}}^2 \simeq (2.5_{-0.9}^{+1.4}) \times 10^{-3}$  eV<sup>2</sup>, respectively. With SNO[2] and KamLAND[3], data have narrowed down the possible mass spectrum of neutrinos into two types, normal hierarchy ( $m_1 \lesssim m_2 < m_3$ ) and inverse hierarchy ( $m_3 < m_1 \lesssim m_2$ ). If the experimental results  $\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2$  and  $\Delta m_{\text{atm}}^2 = |m_3^2 - m_2^2|$  are accommodated to the masses of normal hierarchy, one can obtain the following relations for mass ratio,

$$\frac{m_2}{m_3} \approx \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} = (1.7_{-0.6}^{+1.7}) \times 10^{-1}, \quad (1)$$

assuming  $m_1$  is strongly restricted to be smaller than  $m_2$  by the order of magnitude, rather than  $m_1 \lesssim m_2$ .

The upper bound  $\sin^2 2\theta_{13} < 0.2$  was obtained from the Chooz experiment[4]. At the present stage in neutrino physics, the yet-unknown  $\theta_{13}$  is considered as the key element to shed light on the feasibility of CP violation and the mass hierarchy. A future long-baseline experiment proposed currently intends to improve the sensitivity up to  $\sin^2 2\theta_{13} < 0.05$  in combination with a reactor experiment, and further  $\sin^2 2\theta_{13} < 0.02$  as beam rates get enhanced[5].

Including data from SNO[2] and KamLand[3], the range of the magnitude of the MNS mixing matrix[6] at the 90% confidence level is given by [7],

$$|U| = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix}, \quad (2)$$

where the unitary mixing matrix is defined via  $\nu_a = \sum_{j=1}^3 U_{aj} \nu_j$  ( $a = e, \mu, \tau$ ), with a flavor eigenstate  $\nu_a$  and a mass eigenstate  $\nu_j$ . It can be readily recognized that the central values of elements in the mixing matrix in Eq.(2) are pointing likely numbers,  $\sin \theta_{sol} = \frac{1}{\sqrt{3}}$  and  $\sin \theta_{atm} = \frac{1}{\sqrt{2}}$  [8][9].

The purpose of this work is to find the relation between dirac phase and majorana phase eligible to the compatibility of the high energy lepton asymmetry to explain the baryon asymmetry in universe and the smallness of  $\theta_{13}$ . The former was estimated via Cosmic Microwave Background and the later is about to be explored in near-future experiments with higher precision[5]. As to be shown, when both types of CP violating phases, Dirac and Majorana, are considered, the lepton asymmetry, in general, does not approach to zero as  $\theta_{13}$  does so. The canonical seesaw mechanism with two right-handed neutrinos will be exploited for the connection of the phenomena in different scales. When we utilize the seesaw mechanism in bottom-up direction, the input parameters are set up from the observed results, and the prediction from a model constructed in Ref.[10] such as

$$\sqrt{2} \sin \theta_{13} = \frac{2}{3} \frac{m_2}{m_3} \quad (3)$$

$$\frac{M_1}{M_2} \approx 2 \times 10^{-4}. \quad (4)$$

If one takes the estimation for  $m_2/m_3$  in Eq.(1), the prediction in Eq.(3) corresponds to  $\sin^2 2\theta_{13} = 0.0255$ , which can be tested in reactor and long-baseline experiment with enhanced sensitivity. Furthermore, a somehow smaller value of  $\sin \theta_{13}$  by a factor 0.8 or 0.6, which can be tested with higher resolution in future experiments, will be analyzed together. The ratio in Eq.(4) was obtained by the same breaking parameter from  $U(1) \times Z_3 \times Z_2$  flavor symmetry by the same mechanism, as the small angle in Eq.(3) was obtained. So, the test of the model giving rise to a certain ratio of  $M_1/M_2$  in Eq.(4) can be done, once  $\sin \theta_{13}$  is measured in experiment. The testability motivates us to take them as inputs in this analysis.

The following contents are organized in order. In Section II, the mass matrix of light neutrinos is given in terms of low energy measurable parameters. In Section III, possible neutrino Yukawa matrices with two right-handed neutrinos are derived through bottom-up seesaw mechanism. The magnitude of CP asymmetry in decays of heavy Majorana neutrinos will be estimated as well. In Section IV, the lepton asymmetry to produce the baryon asymmetry in the universe via sphaleron process is evaluated from the CP asymmetry

and the dilution factor which are parameterized in terms of low energy CP phases and the small mixing angle  $\theta_{13}$ . With respect to different values of  $\theta_{13}$ 's, the  $\delta$ - $\varphi$  space is tested for the amount of lepton asymmetry dictated by the observed baryon asymmetry. For a particular case in which the CP asymmetry in decays of heavy neutrinos or the dilution factor has strong dependence on the mixing angle  $\theta_{13}$ , some consideration on the correlation of the lepton asymmetry to the size of low energy CP violation and that to the amplitude of neutrinoless double beta decay follows as a remark in the last section.

## II. MASSES AND MIXING ANGLES OF LIGHT NEUTRINOS

In general, a unitary mixing matrix for 3 generations of neutrinos is given by

$$\tilde{U} = R(\theta_{23}) R(\theta_{13}, \delta) R(\theta_{12}) P(\varphi, \varphi') \quad (5)$$

where  $R$ 's are rotations with three angles and a Dirac phase  $\delta$  and the  $P = \text{Diag}(1, e^{i\varphi/2}, e^{i\varphi'/2})$  with Majorana phases  $\varphi$  and  $\varphi'$  is a diagonal phase transformation. The mass matrix of light neutrinos is given by  $M_\nu = \tilde{U} \text{Diag}(m_1, m_2, m_3) \tilde{U}^T$ , where  $m_1, m_2, m_3$  are the real positive masses of light neutrinos. Or the Majorana phases can be embedded in the diagonal mass matrix such that

$$M_\nu = U \text{Diag}(m_1, \check{m}_2, \check{m}_3) U^T, \quad (6)$$

where  $U \equiv \tilde{U} P^{-1}$  and  $\check{m}_2 \equiv m_2 e^{i\varphi}$  and  $\check{m}_3 \equiv m_3 e^{i\varphi'}$ . If transformation matrix  $U$  in the standard parametrization for CKM [11] is

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (7)$$

where  $s_{ij}$  and  $c_{ij}$  denotes  $\sin \theta_{ij}$  and  $\cos \theta_{ij}$  with the mixing angle  $\theta_{ij}$  between  $i$ -th and  $j$ -th generations, and if  $s_{12} = \frac{1}{\sqrt{3}}(1 + \sigma)$ ,  $\sigma \ll 1$  and  $s_{23} = \frac{1}{\sqrt{2}}$ ,  $s_{13} \ll 1$  are adopted for the angles in the transformation  $U$ , the matrix  $M_\nu$  can be expressed as

$$M_\nu \approx \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{\check{m}_2}{3} \begin{pmatrix} 1 + 2\sigma & 1 + \frac{1}{2}\sigma & 1 + \frac{1}{2}\sigma \\ & 1 - \sigma & 1 - \sigma \\ & & 1 - \sigma \end{pmatrix} + \frac{\check{m}_3}{2} \begin{pmatrix} 2\vartheta^2 & -\sqrt{2}\vartheta & \sqrt{2}\vartheta \\ & 1 - \vartheta^2 & -1 + \vartheta^2 \\ & & 1 - \vartheta^2 \end{pmatrix} \quad (8)$$

where  $\vartheta = s_{13}e^{-i\delta}$  with a Dirac phase  $\delta$ .

Inspired from the form of mass in Eq.(8), an ansatz for mass matrix of neutrinos is introduced by adopting the leading contributions such that,

$$\frac{M_\nu}{m^*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \rho e^{i\varphi} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \theta e^{-i\delta} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (9)$$

which is divided by  $m^*$  of mass dimension. It was shown that the above structure of neutrino mass could be derived using a flavor symmetry[10]. We will take a value  $\rho = 0.11$  that corresponds to  $(2/3)$  times  $\sqrt{\Delta m_{sol}^2/\Delta m_{atm}^2}$  given in Eq.(1). Regarding  $\theta$  that corresponds to  $\sqrt{2} \sin \theta_{13}$ , we take its upper bound to be  $\rho$ , such as  $\theta \leq \rho$ . When the  $\theta$  varies from 0 to  $\rho$ , the dimensionless matrix in Eq.(9) gives rise to the magnitude of transformation with such ranges as

$$\begin{pmatrix} 0.816 - 0.825 & 0.577 - 0.560 & 0 - 0.0821 \\ 0.408 - 0.352 & 0.577 - 0.622 & 0.707 - 0.700 \\ 0.408 - 0.443 & 0.577 - 0.548 & 0.707 - 0.710 \end{pmatrix}, \quad (10)$$

while the mass ratios take ranges as  $m_2/m_3 = 0.165 - 0.162$  and  $m_1/m_3 = 1 \times 10^{-10} - 4 \times 10^{-3}$ . The ranges should be read as a value at  $\theta = 0$  to that at  $\theta = \rho$ . Thus, it is clear that the smallest mass eigenvalue can be taken as zero and accordingly only a single relative Majorana phase as in Eq.(9) can be chosen without loss of generality.

When  $\theta = \rho$ , the Jarlskog invariant  $\mathcal{J} = \text{Im}[U_{e1}U_{\tau3}U_{\tau1}^*U_{e3}^*]$  of the transformation of the mass matrix in Eq.(9) can be evaluated using Eq.(7) and Eq.(10). The  $s_{12}s_{23}$ , the first term in  $U_{\tau1}$  is simply  $U_{\tau1}$  if  $s_{13} = 0$ . So, by taking the sizes of  $U_{e1}, U_{\tau3}, U_{e3}$  at  $\theta = \rho$  and that of  $U_{\tau1}$  at  $\theta = 0$ , we can obtain the magnitude of CP violation as

$$\mathcal{J} = |U_{e1}U_{e3}U_{\tau3}|_{\theta=\rho}|U_{\tau1}|_{\theta=0} \sin \delta, \quad (11)$$

whose measurement is possible from neutrino oscillations. Although Majorana CP violating phases are not detectable through neutrino oscillations, they may affect the amplitude of neutrinoless double beta decay  $\langle m_{ee} \rangle \equiv |\sum_{i=1}^3 U_{ei}^2 m_i e^{i\varphi_i}|$  where  $\varphi_i$  denotes the Majorana phases. The amplitude with a single Majorana phase from the ansatz in Eq.(9) and the

numerical values in Eq.(10) is

$$\langle m_{ee} \rangle = m_3 \sqrt{\rho^2 |U_{e2}|^2 + |U_{e3}|^2 + 2\rho |U_{e2}U_{e3}| \cos(\varphi - 2\delta)}, \quad (12)$$

if one of the masses is considered as zero so that the number of physical Majorana phases is one.

In the following sections, the parameters are set up such that two mass eigenvalues and two large mixing angles are fixed based on experimental data while two CP phases are still free with respect to light neutrinos. The small angle  $\theta_{13}$  and the ratio  $M_1/M_2$  of heavy right neutrinos are taken as in Eq.(3) and Eq.(4) predicted from a particular model [10], while  $M_2$  is chosen to be of the scale of Grand Unified Theory. As for  $\theta_{13}$ , the value for  $\theta = 0.6\rho$  involves the lepton asymmetry curve of which extremum is about that of the observed baryon asymmetry to be shown in the last section, so that the value showing such aspect will be considered as well as the value of Eq.(3) for comparison.

### III. SEESAW MECHANISM AND NEUTRINO YUKAWA MATRICES

Neutrino mass models with one zero mass involved in three active neutrinos can be generated naturally from the seesaw mechanism with two right-handed neutrinos. In the basis mass matrix  $M_R$  of right-handed neutrinos  $N_R = (N_1, N_2)$  is diagonal, a model is given in terms of Yukawa interactions of leptons and lepton number violating Majorana mass  $M_R$  of right-handed neutrinos:

$$-\mathcal{L} = H\mathcal{Y}_\ell L_e \bar{e}_R + H\mathcal{Y}_\nu L_e \bar{N}_R + \frac{1}{2}M_R N_R N_R, \quad (13)$$

which consists of a  $3 \times 3$  matrix  $\mathcal{Y}_\ell$ , a  $3 \times 2$  matrix  $\mathcal{Y}_\nu$  a  $2 \times 2$  matrix  $M_R$ , with two right-handed neutrinos. When Yukawa interaction of neutrinos is introduced, the light masses are derived through the seesaw mechanism[12],  $M_\nu = -v^2 \mathcal{Y}_\nu M_R^{-1} \mathcal{Y}_\nu^T$  in top-down approach. On the other hand, the matrix  $\mathcal{Y}_\nu$  is found as the solution to the seesaw mechanism in bottom-up approach once one launches the analysis with the light neutrino masses  $M_\nu$ . Let  $M_1$  and  $M_2$  be the masses of right-handed neutrinos and  $M_{ij}$  the elements of the matrix  $M_\nu$ . If Yukawa matrix is given as,

$$\mathcal{Y}_\nu = \begin{pmatrix} \sqrt{\mu}a_1 & b_1 \\ \sqrt{\mu}a_2 & b_2 \\ \sqrt{\mu}a_3 & b_3 \end{pmatrix}, \quad (14)$$

where  $\sqrt{\mu} \equiv \sqrt{M_1/M_2}$ . The elements  $a_j$ 's and  $b_j$ 's are distinguished such that only one of the  $b_j$ 's is zero, and then a matrix  $\mathcal{Y}_\nu = (\sqrt{\mu}b_j \ a_j)$  also will be used when a texture zero is placed in the first column. One can obtain its entries in terms of dimensionless  $\tilde{M}_{jk} \equiv M_{jk}/m^*$  with  $j, k = 1, 2, 3$ ,

$$\begin{aligned} a_1 &= \sqrt{\tilde{M}_{11} - b_1^2}, \text{ or } b_1 = \sqrt{\tilde{M}_{11} - a_1^2}, \\ a_i &= \frac{1}{\tilde{M}_{11}} \left[ a_1 \tilde{M}_{1i} - \sigma_i b_1 \sqrt{\tilde{M}_{11} \tilde{M}_{ii} - \tilde{M}_{1i}^2} \right], \\ b_i &= \frac{1}{\tilde{M}_{11}} \left[ b_1 \tilde{M}_{1i} + \sigma_i a_1 \sqrt{\tilde{M}_{11} \tilde{M}_{ii} - \tilde{M}_{1i}^2} \right], \\ \tilde{M}_{11} \tilde{M}_{23} &= \left[ \tilde{M}_{12} \tilde{M}_{13} + \sigma_2 \sigma_3 \sqrt{(\tilde{M}_{11} \tilde{M}_{22} - \tilde{M}_{12}^2)(\tilde{M}_{11} \tilde{M}_{33} - \tilde{M}_{13}^2)} \right]. \end{aligned} \quad (15)$$

where the  $i$  is 2 or 3, the  $\sigma_i$  is a sign  $\pm$ , and the sign of  $a_1$  is fixed as positive. However, the relative sign of  $a_1$  to  $b_1$  is still undetermined. The above expression derived in Ref.[13] is equivalent to the neutrino mass matrix in the following way:

$$M_\nu = \frac{v^2}{M_2} \begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 + b_1 b_2 & a_1 a_3 + b_1 b_3 \\ a_1 a_2 + b_1 b_2 & a_2^2 + b_2^2 & a_2 a_3 + b_2 b_3 \\ a_1 a_3 + b_1 b_3 & a_2 a_3 + b_2 b_3 & a_3^2 + b_3^2 \end{pmatrix} \quad (16)$$

It is clear that only 5 out of 6 parameters  $(a_1, b_1, a_i, b_i)$  can be specified in terms of the elements of  $M_\nu$ , since 6 independent elements of the symmetric matrix are related by the zero determinant. There are various ways to decrease the number of parameters in a Yukawa matrix, posing a texture zero or posing equalities between elements for the matrix texture. It is known that texture zeros or equalities among matrix entries can be generated by imposing additional symmetries to the theory. In this work, we focus on the types of Yukawa matrices with a texture zero.

If one takes the following matrix introduced in Eq.(9)

$$\begin{pmatrix} \rho e^{i\varphi} & \rho e^{i\varphi} - \theta e^{-i\delta} & \rho e^{i\varphi} + \theta e^{-i\delta} \\ \rho e^{i\varphi} - \theta e^{-i\delta} & \rho e^{i\varphi} + 1 & \rho e^{i\varphi} - 1 \\ \rho e^{i\varphi} + \theta e^{-i\delta} & \rho e^{i\varphi} - 1 & \rho e^{i\varphi} + 1 \end{pmatrix} \quad (17)$$

for  $\tilde{M}_{jk}$  used in Eq.(15), it will give rise to the 6 possible matrices for  $\mathcal{Y}_\nu$  in Eq.(14) which consist of the three with one of Yukawa couplings of  $N_2$  absent;

$$\begin{aligned}
\mathcal{Y}[y_{12} = 0] &= \frac{1}{\sqrt{\rho e^{i\varphi}}} \begin{pmatrix} \sqrt{\mu} \rho e^{i\varphi} & 0 \\ \sqrt{\mu} (\rho e^{i\varphi} - \theta e^{-i\delta}) & \sqrt{\rho e^{i\varphi} (\rho e^{i\varphi} + 1) - (\rho e^{i\varphi} - \theta e^{-i\delta})^2} \\ \sqrt{\mu} (\rho e^{i\varphi} + \theta e^{-i\delta}) & -\sqrt{\rho e^{i\varphi} (\rho e^{i\varphi} + 1) - (\rho e^{i\varphi} + \theta e^{-i\delta})^2} \end{pmatrix}, \\
\mathcal{Y}[y_{22} = 0] &= \frac{1}{\sqrt{\rho e^{i\varphi} + 1}} \begin{pmatrix} \sqrt{\mu} (\rho e^{i\varphi} - \theta e^{-i\delta}) & -\sqrt{\rho e^{i\varphi} (\rho e^{i\varphi} + 1) - (\rho e^{i\varphi} - \theta e^{-i\delta})^2} \\ \sqrt{\mu} (\rho e^{i\varphi} + 1) & 0 \\ \sqrt{\mu} (\rho e^{i\varphi} - 1) & \sqrt{(\rho e^{i\varphi} + 1)^2 - (\rho e^{i\varphi} - 1)^2} \end{pmatrix} \quad (18) \\
\mathcal{Y}[y_{32} = 0] &= \frac{1}{\sqrt{\rho e^{i\varphi} + 1}} \begin{pmatrix} \sqrt{\mu} (\rho e^{i\varphi} + \theta e^{-i\delta}) & \sqrt{\rho e^{i\varphi} (\rho e^{i\varphi} + 1) - (\rho e^{i\varphi} + \theta e^{-i\delta})^2} \\ \sqrt{\mu} (\rho e^{i\varphi} - 1) & -\sqrt{(\rho e^{i\varphi} + 1)^2 - (\rho e^{i\varphi} - 1)^2} \\ \sqrt{\mu} (\rho e^{i\varphi} + 1) & 0 \end{pmatrix},
\end{aligned}$$

and three more matrices,  $\mathcal{Y}[y_{11} = 0]$ ,  $\mathcal{Y}[y_{21} = 0]$ , and  $\mathcal{Y}[y_{31} = 0]$ , which make the counterparts to the above three sets by exchanging their columns, that is, when one of Yukawa couplings of  $N_1$  is absent.

Once we have the neutrino Yukawa matrix, we can calculate the magnitude of CP asymmetry  $\epsilon_i$  in decays of heavy Majorana neutrinos [14, 15],

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow \ell H) - \Gamma(N_i \rightarrow \bar{\ell} H^*)}{\Gamma(N_i \rightarrow \ell H) + \Gamma(N_i \rightarrow \bar{\ell} H^*)}, \quad (19)$$

where  $i$  denotes a generation. When one of two generations of right neutrinos has a mass far below that for the other generation, i.e.,  $M_1 \ll M_2$ , the  $\epsilon_i$  in Eq.(19) can be replaced by just  $\epsilon_1$  obtained from the decay of  $M_1$  [14, 15],

$$\epsilon_1 = \frac{1}{8\pi} \frac{\text{Im}[(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{12}^2]}{(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{11}} f\left(\frac{M_2}{M_1}\right), \quad (20)$$

where  $f(M_2/M_1)$  represents loop contribution to the decay width from vertex and self energy and is given by

$$f(x) = x \left[ 1 - (1 + x^2) \ln \frac{1 + x^2}{x^2} + \frac{1}{1 - x^2} \right] \quad (21)$$

for the Standard Model. For a large value of  $x$ , the leading order of  $f(x)$  is of  $3x^{-1}/2$ . It is convenient to consider separately the factor that depends on Yukawa matrix in  $\epsilon_1$  in Eq.(20) at this stage.

$$\frac{\text{Im}[(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{12}^2]}{(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{11}} = \frac{\text{Im}[(y_{11}^* y_{12} + y_{21}^* y_{22} + y_{31}^* y_{32})^2]}{|y_{11}|^2 + |y_{21}|^2 + |y_{31}|^2} \equiv \Delta_1, \quad (22)$$

$$\epsilon_1 = \frac{3}{16\pi} \Delta_1 \frac{M_1}{M_2}, \quad (23)$$

for  $M_1 \ll M_2$ , where  $a$ 's and  $b$ 's are defined in Eq.(11). The ratio  $M_1/M_2 \sim 2 \times 10^{-4}$  in Eq.(4) will be used for the evaluation of the CP asymmetry.

From a number of types of matrices with a texture zero derived in Eq.(18), 6 different values of  $\Delta_1$ 's can be evaluated and expressed using the following polynomials:

$$\mathcal{A}_1(\rho^0), \mathcal{A}_2(\rho^0) \equiv 2 + 3\rho^2 \mp 2\rho\theta \cos(\varphi + \delta) + \theta^2 \quad (24)$$

$$\mathcal{B}_1(\rho), \mathcal{B}_2(\rho) \equiv \sqrt{\rho^2 \pm 4\rho^2\theta \cos \delta - 2\rho\theta^2 \cos(\varphi + 2\delta) + 4\rho^2\theta^2 \mp 4\rho\theta^3 \cos(\varphi + \delta) + \theta^4} \quad (25)$$

where  $\mathcal{A}_1(\rho^0)$  in Eq.(24) denotes the terms with the upper sign and indicates that the leading term in  $\mathcal{A}_1$  is of order  $\rho^0$ . So does  $\mathcal{A}_2$  with lower sign. Likewise,  $\mathcal{B}_1$  of leading order  $\rho$  in Eq.(25) denotes the terms with the upper sign inside the square root. If  $y_{11}$  and  $y_{12}$  is zero, one can obtain the following expression,

$$|a_1|^2 + |a_2|^2 + |a_3|^2 = \rho^{-1}(3\rho^2 + 2\theta^2), \quad (26)$$

$$|b_1|^2 + |b_2|^2 + |b_3|^2 = \rho^{-1}\{\mathcal{B}_1(\rho) + \mathcal{B}_2(\rho)\} \quad (27)$$

$$Im[(a_1^*b_1 + a_2^*b_2 + a_3^*b_3)^2] = C_0 + C_1\theta + C_2\theta^2 + \mathcal{O}(\theta^3), \quad (28)$$

$$C_0 = 0,$$

$$C_1 = 0,$$

$$C_2 = 4\rho^{-1} \sin(\varphi + 2\delta) + \mathcal{O}(\rho).$$

If one of  $y_{j2}$ 's is zero,  $\sum |y_{j1}|^2 = \sum |a_j|^2$ ,  $\sum |y_{j2}|^2 = \sum |b_j|^2$ ,  $Im[(y_{j1}^*y_{j2})^2] = Im[(a_j^*b_j)^2]$ , while, if one of  $y_{j1}$ 's is zero,  $\sum |y_{j1}|^2 = \sum |b_j|^2$ ,  $\sum |y_{j2}|^2 = \sum |a_j|^2$ ,  $Im[(y_{j1}^*y_{j2})^2] = -Im[(a_j^*b_j)^2]$ . Thus, the CP asymmetry for  $y_{11} = 0$  or  $y_{12} = 0$  is proportional to

$$\Delta_1[y_{11}, y_{12} = 0] = \left[ \frac{1}{\mathcal{B}_1(\rho) + \mathcal{B}_2(\rho)}, \frac{-1}{3\rho^2 + \theta^2} \right] \{4\theta^2 \sin(\varphi + 2\delta) + \mathcal{O}(\rho^2\theta^2)\}, \quad (29)$$

respectively. The approximation in Eq.(29) is valid unless  $\pi - \rho < \varphi < \pi + \rho$ .

When  $y_{21}$  or  $y_{31}$  is zero or when  $y_{22}$  or  $y_{32}$  is zero can be considered in terms of the following expressions;

$$|a_1|^2 + |a_2|^2 + |a_3|^2 = \sqrt{\rho^2 + 2\rho \cos \varphi + 1}^{-1} \begin{cases} \mathcal{A}_1(\rho^0) & \text{for } [b_2 = 0] \\ \mathcal{A}_2(\rho^0) & \text{for } [b_3 = 0] \end{cases} \quad (30)$$

$$|b_1|^2 + |b_2|^2 + |b_3|^2 = \sqrt{\rho^2 + 2\rho \cos \varphi + 1}^{-1} \left( 4\rho + \begin{cases} \mathcal{B}_1(\rho) & \text{for } [b_2 = 0] \\ \mathcal{B}_2(\rho) & \text{for } [b_3 = 0] \end{cases} \right) \quad (31)$$

$$\text{Im}[(a_1^*b_1 + a_2^*b_2 + a_3^*b_3)^2] = (\rho^2 + 2\rho \cos \varphi + 1)^{-1} \{C_0 + C_1\theta + C_2\theta^2 + \mathcal{O}(\theta^3)\}, \quad (32)$$

$$\begin{aligned} C_0 &= 4\rho \sin \varphi - 4\rho^2 \sin(2\varphi) + \mathcal{O}(\rho^5), \\ C_1 &= \mp 4\rho \sin(\delta + \varphi) + \mathcal{O}(\rho^2), \\ C_2 &= \mathcal{O}(\rho), \end{aligned} \quad (33)$$

where  $C_1$  takes  $-$  sign if  $b_2 = 0$ , and  $+$  sign if  $b_3 = 0$ . Thus, the CP asymmetry is proportional to

$$\Delta_1 [y_{21}, y_{31} = 0] = \frac{\sqrt{\rho^2 + 2\rho \sin \varphi + 1}}{4\rho + [\mathcal{B}_1(\rho), \mathcal{B}_2(\rho)]} \{C_0(\rho) + C_1(\rho)\theta + C_2(\rho)\theta^2\}, \quad (34)$$

$$\Delta_1 [y_{22}, y_{32} = 0] = -\frac{\sqrt{\rho^2 + 2\rho \sin \varphi + 1}}{[\mathcal{A}_1(\rho^0), \mathcal{A}_2(\rho^0)]} \{C_0(\rho) + C_1(\rho)\theta + C_2(\rho)\theta^2\}. \quad (35)$$

The CP asymmetry in Eq.(23) is clearly parameterized by the ratio of heavy neutrino masses and  $\Delta_1$  expressed fully by low-energy observables. Without  $\phi$ , the CP asymmetry depends on  $\theta_{13}^2 \sin(2\delta)$  in the leading order if  $y_{11}$  or  $y_{12} = 0$ , and  $\theta_{13} \sin \delta$  otherwise. On the other hand, the CP asymmetry without  $\theta_{13}$  depends on  $\sin \varphi$  as shown in Ref.[9]. A number of models were discussed about in focus of the connection of measurable CP violations in low energy and the CP asymmetry for the leptogenesis[16].

#### IV. LEPTON ASYMMETRY

The baryon density of our universe  $\Omega_B h^2 = 0.0224 \pm 0.0009$  implied by WMAP (Wilkinson Microwave Anisotropy Probe) data indicates the observed baryon asymmetry in the Universe[17],

$$\eta_B^{CMB} = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.5_{-0.3}^{+0.4}) \times 10^{-10}, \quad (36)$$

where  $n_B, n_{\bar{B}}$  and  $n_\gamma$  are number density of baryon, anti-baryon and photon, respectively. The leptogenesis [18] has become a compelling theory to explain the observed baryon asym-

metry in the universe, due to increasing reliance on the seesaw mechanism from experiments. The baryon asymmetry Eq.(36) can be rephrased

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \simeq (8.8 - 9.8) \times 10^{-11}. \quad (37)$$

The  $n_\gamma$  is the photon number density and the  $s$  is entropy density so that the number density with respect to a co-moving volume element is taken into account. The baryon asymmetry produced through sphaleron process is related to the lepton asymmetry [19] by  $Y_B = \frac{a}{a-1} Y_L$  with  $a \equiv (8N_F + 4N_H)/(22N_F + 13N_H)$ , for example,  $a = 28/79$  for the Standard Model(SM) with three generations of fermions and a single Higgs doublet,  $N_F = 3, N_H = 1$ . The generation of a lepton asymmetry requires the CP-asymmetry and out-of-equilibrium condition. The  $Y_L$  is explicitly parameterized by two factors,  $\epsilon$ , the size of CP asymmetry, and  $\kappa$ , the dilution factor from washout process.

$$Y_L = \frac{(n_L - n_{\bar{L}})}{s} = \kappa \frac{\epsilon_i}{g^*} \quad (38)$$

where  $g^* \simeq 110$  is the number of relativistic degree of freedom.

The  $\kappa$  in Eq.(38) is determined by solving the full Boltzmann equations. The  $\kappa$  can be simply parameterized in terms of  $K$  defined as the ratio of  $\Gamma_1$  the tree-level decay width of  $\nu_{R1}$  to  $H$  the Hubble parameter at temperature  $M_1$ , where  $K \equiv \Gamma_1/H < 1$  describes processes out of thermal equilibrium and  $\kappa < 1$  describes washout effect[19],

$$\kappa = \frac{0.3}{K (\ln K)^{0.6}} \text{ for } 10 \lesssim K \lesssim 10^6, \quad (39)$$

$$\kappa = \frac{1}{2\sqrt{K^2 + 9}} \text{ for } 0 \lesssim K \lesssim 10. \quad (40)$$

The decay width of  $N_1$  by the Yukawa interaction at tree level and Hubble parameter in terms of temperature  $T$  and the Planck scale  $M_{pl}$  are  $\Gamma_1 = (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{11} M_1/(8\pi)$  and  $H = 1.66g_*^{1/2} T^2/M_{pl}$ , respectively. At temperature  $T = M_1$ , the ratio  $K$  is

$$K = \frac{M_{pl}}{1.66\sqrt{g^*}(8\pi)} \frac{(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{11}}{M_1}, \quad (41)$$

which reduces to, since  $(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{11} = \mu \sum |a_i|^2$  in Eq.(14),

$$K = \frac{M_{pl}/M_2}{1.66\sqrt{g^*}(8\pi)} \sum |a_i|^2, \quad (42)$$

or which can be proportional to  $\sum |b_i|^2$  depending on the position of zero. For the evaluation of  $K$ ,  $M_2 \approx M_{GUT}$  would be taken.

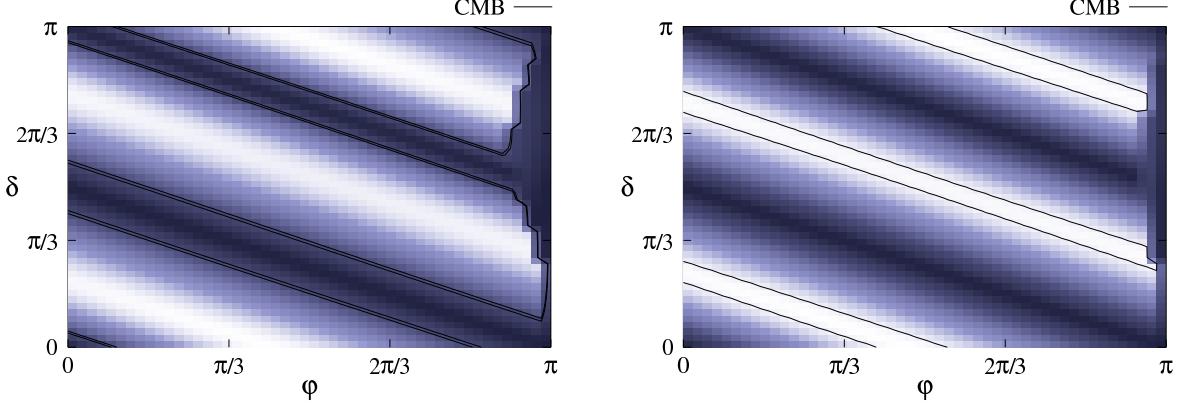


FIG. 1:  $Y_L$  vs.  $\delta$  and  $\varphi$  when  $y_{11} = 0$  (a)  $\theta = \rho$ , (b)  $\theta = 0.6\rho$

The washout effect of the asymmetry varies significantly depending on the structure of Yukawa matrix, i.e., when the decay width is determined by each type of Yukawa matrix listed in Eq.(18) and three more with a texture zero in counter-position. Out of all the types of Yukawa matrices examined, there is no such a case that Yukawa couplings originate decays of neutrinos  $N_1$  which satisfy the out-of-equilibrium condition  $K < 1$  at  $T = M_1$ . The dilution factor  $\kappa$  given in Eq.(39) and Eq.(40) changes in size when it is described in terms of low energy phases  $\delta$  and  $\varphi$ , although the variation in dilution factor is not so remarkable as to affect the order of magnitude as follows. If  $y_{11}$  is 0,  $\kappa = (6.9 - 8.9) \times 10^{-3}$  as  $(\delta, \varphi)$  varies  $(\pi, \pi)$  to  $(0, 0)$ . If  $y_{12}$  is 0,  $\kappa = 8.0 \times 10^{-2}$  and the value does not vary on  $\delta$  and  $\varphi$ . If  $y_{21}$  is 0,  $\kappa = (6.9 - 9.1) \times 10^{-2}$  as  $(\delta, \varphi)$  varies  $(0, \pi)$  to  $(\pi, 0)$ . If  $y_{22}$  is 0,  $\kappa = (6.5 - 8.7) \times 10^{-3}$  as  $(\delta, \varphi)$  varies  $(0, \pi)$  to  $(0, 0)$ . If  $y_{31}$  is 0,  $\kappa = (6.9 - 9.1) \times 10^{-2}$  as  $(\delta, \varphi)$  varies  $(0, 0)$  to  $(\pi, \pi)$ . And if  $y_{32}$  is 0,  $\kappa = (6.5 - 8.7) \times 10^{-3}$  as  $(\delta, \varphi)$  varies  $(\pi, \pi)$  to  $(\pi, 0)$ . The dilution factor is enhanced most with the Yukawa matrix of type  $[y_{12} = 0]$ ,  $[y_{21} = 0]$  or  $[y_{31} = 0]$ , which shows that the amount of asymmetry survived from washout is at most 10%. It is worth reminding that the survived portion of asymmetry cannot exceed 17% even if one took  $K = 0$ . The rest three types of Yukawa couplings give rise to even lower dilution factor. When  $T < M_1$ , the Boltzmann equations still depict the finite value of  $\kappa$  as  $M_1/T$  increases for the universe evolution [14][15].

The amount of lepton asymmetry in Eq.(38) is now given as a function of  $\delta$  and  $\varphi$  as well as  $\theta_{13}$ ,

$$Y_L = \frac{1}{g^*} \kappa(\delta, \varphi, \theta_{13}) \epsilon_1(\delta, \varphi, \theta_{13}), \quad (43)$$

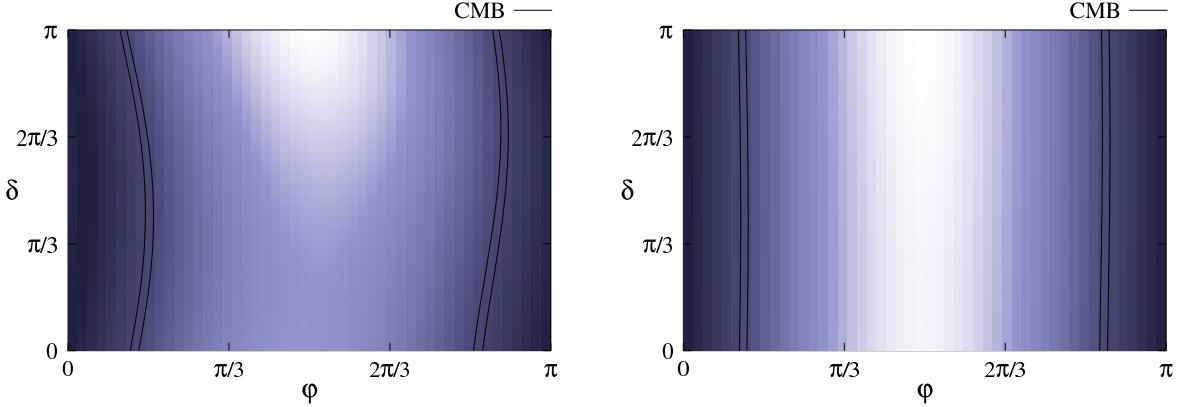


FIG. 2:  $Y_L$  vs.  $\delta$  and  $\varphi$  when  $y_{22} = 0$  (a)  $\theta = \rho$ , (b)  $\theta = 0.1\rho$

which imbeds  $\Delta_1$ 's analyzed in the last section and  $K$ 's in Eq.(42). The Fig.1 and Fig.2 show the dependence of  $Y_L$  on the Dirac phase  $\delta$  and the Majorana phase  $\varphi$ . The brighter part represents a region of higher  $Y_L$ , while the darker represents that of lower  $Y_L$ . The contour is the amount of  $Y_L$  dictated by Cosmic Microwave Background observation [17][19]. Hereafter, it will be denoted by  $Y_L^{CMB}$ . The dark region outside of the contour  $Y_L^{CMB}$  in  $\delta$ - $\varphi$  space is ruled out. In Fig.1, the pattern of hue and the contour of  $Y_L^{CMB}$  manifest the proportionality of the obtained  $Y_L$  to  $\sin(\varphi + 2\delta)$  as analyzed in Eq.(28).

When  $\theta = \rho$  as in Fig.1 (a), the sufficient amount of lepton asymmetry prevails in most part of  $\delta$ - $\varphi$  space. When  $\theta = 0.6\rho$  as in Fig.1 (b), only strongly restricted region just above  $Y_L^{CMB}$  can be regarded as plausible for leptogenesis. In Fig.2 for  $y_{21} = 0$ , it shows clearly that the size of angle  $\theta_{13}$  does not affect the amount of the lepton asymmetry. The shape of the hue in Fig.1 and Fig.2 can be shown by their cross-sections as in Fig.3. In Fig.3 (a) with  $y_{11} = 0$ , the Majorana phase is strongly constrained by the Dirac phase, while the  $Y_L$  vs.  $\varphi$  relation does not rely on  $\delta$  so significantly in Fig.3 (b) with  $y_{22} = 0$  as in Fig.3 (a). Aspects of the relation of lepton asymmetry and observable CP phases for the case with  $y_{32} = 0$  are similar to those for the case with  $y_{22} = 0$ , regarding the amount of the asymmetry and the correlation of two types of phases.

For the other three cases with  $y_{12} = 0$ ,  $y_{21} = 0$  and  $y_{31} = 0$ , the amount of lepton asymmetry is enhanced by about two orders of magnitude since washout effect is suppressed and the size of CP asymmetry is increased in comparison with those of the previous three cases. From Fig.3, it is obvious that most range in  $\delta$ - $\varphi$  space is involved with the amount

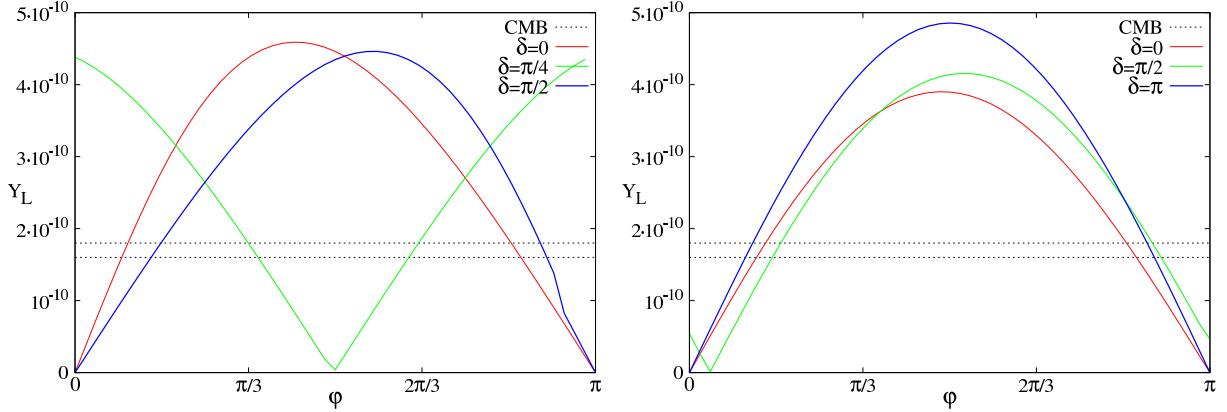


FIG. 3: (color online)  $Y_L$  vs.  $\varphi$  at  $\theta = \rho$ , the cross sections by particular values of  $\delta$ 's of the curves (a) in Fig.1-(a) for  $y_{11} = 0$ , and (b) in Fig.2-(a) for  $y_{22} = 0$ .

of the asymmetry above  $Y_L^{CMB}$ . As for the correlation of two CP phases, the  $\delta$  and  $\varphi$  with  $y_{12} = 0$  also constrain each other by  $\sin(\varphi + 2\delta)$  as so do the phases with  $y_{11} = 0$ . When  $y_{21} = 0$  or  $y_{31} = 0$ , the asymmetry mainly depends on  $\sin \varphi$  as seen for  $y_{22} = 0$  or  $y_{32} = 0$ .

It has been revealed that a Yukawa matrix with a texture zero at  $y_{11} = 0$  or  $y_{12} = 0$  originates the lepton asymmetry strongly correlated with  $\theta_{13}$  and  $\delta$ . The next section includes the discussion on the possible amount of the lepton asymmetry to the size of CP violation in neutrino oscillation and the amplitude of neutrinoless double beta decay.

## V. REMARKS

There has been consideration about the way the amount of the lepton asymmetry is dependent on the low energy measurable  $\theta_{13}$ ,  $\varphi$ , and  $\delta$  in Eq.(9) and how the dependence varies according to structures of Yukawa matrices, even though those different types of Yukawa matrices can cause the same prediction for the low energy parameters. In Fig.4 and Fig.5, the case with Yukawa matrix with a texture zero at  $y_{11} = 0$  is considered for the compatibility of the production of sufficient lepton asymmetry and the possible measurement of Jarlskog invariant of  $\mathcal{J}$  and the amplitude of neutrinoless double beta decay. For the particular case, the figures clearly show that if  $\sin^2 2\theta_{13}$  is smaller than 0.01, the lepton asymmetry obtained from the Yukawa couplings is below the amount of asymmetry based on the baryon asymmetry observation.

If  $\rho$  and  $\theta$  in Eq.(9) has such relations as  $\theta = \rho$ ,  $\theta = 0.8\rho$ , and  $\theta = 0.6\rho$ , each case

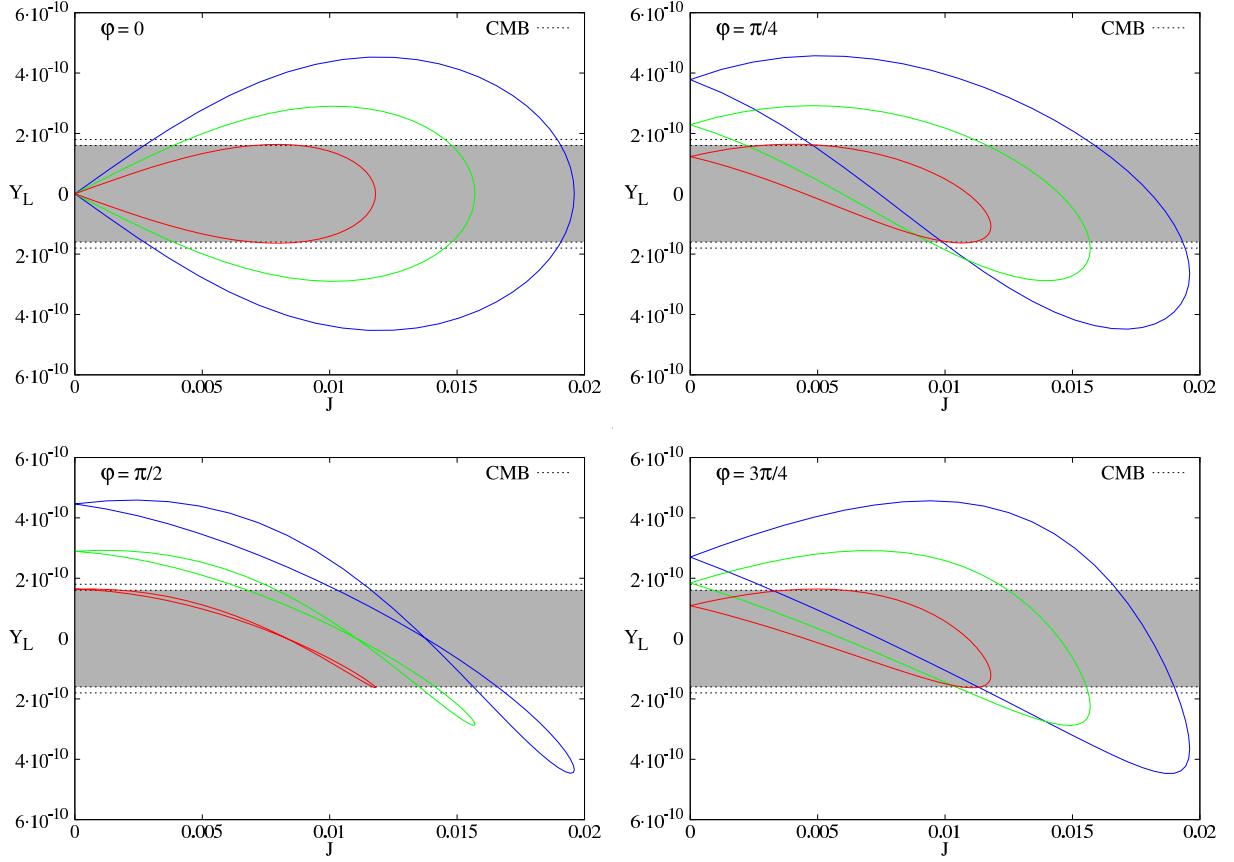


FIG. 4: (color online)  $Y_L$  vs.  $\mathcal{J}$  for chosen values of Majorana phase. The large(blue), medium(green), and small(red) curves correspond to  $\sin^2 2\theta_{13} = 0.0255$ ,  $0.0164$ , and  $0.00922$ , respectively.

corresponds to  $\sin^2 2\theta_{13} = 0.0255$ ,  $0.0164$ , and  $0.00922$ . Although the 5 elements in Eq.(15) were determined by taking a texture zero among themselves, the relative sign of two columns is not yet specified and, accordingly, only the absolute value of the asymmetry is concerned with the Jarlskog invariant as shown in Fig.4.

Once the hierarchy in masses of the right-handed heavy neutrinos is determined, the lepton asymmetry from Yukawa couplings with a texture zero at other than  $y_{11}$  and  $y_{12}$  has main correlation with the Majorana phase  $\varphi$ . The correlation of  $Y_L$  to the small mixing angle  $\theta_{13}$  and the Dirac phase  $\delta$  appears strongly when  $y_{11}$  or  $y_{12}$  is zeroed. In other words, the Dirac phase  $\delta$  and the Majorana phase  $\varphi$  constrains each other so that only a certain region of  $\delta$ - $\varphi$  space can be compatible with the leptogenesis.

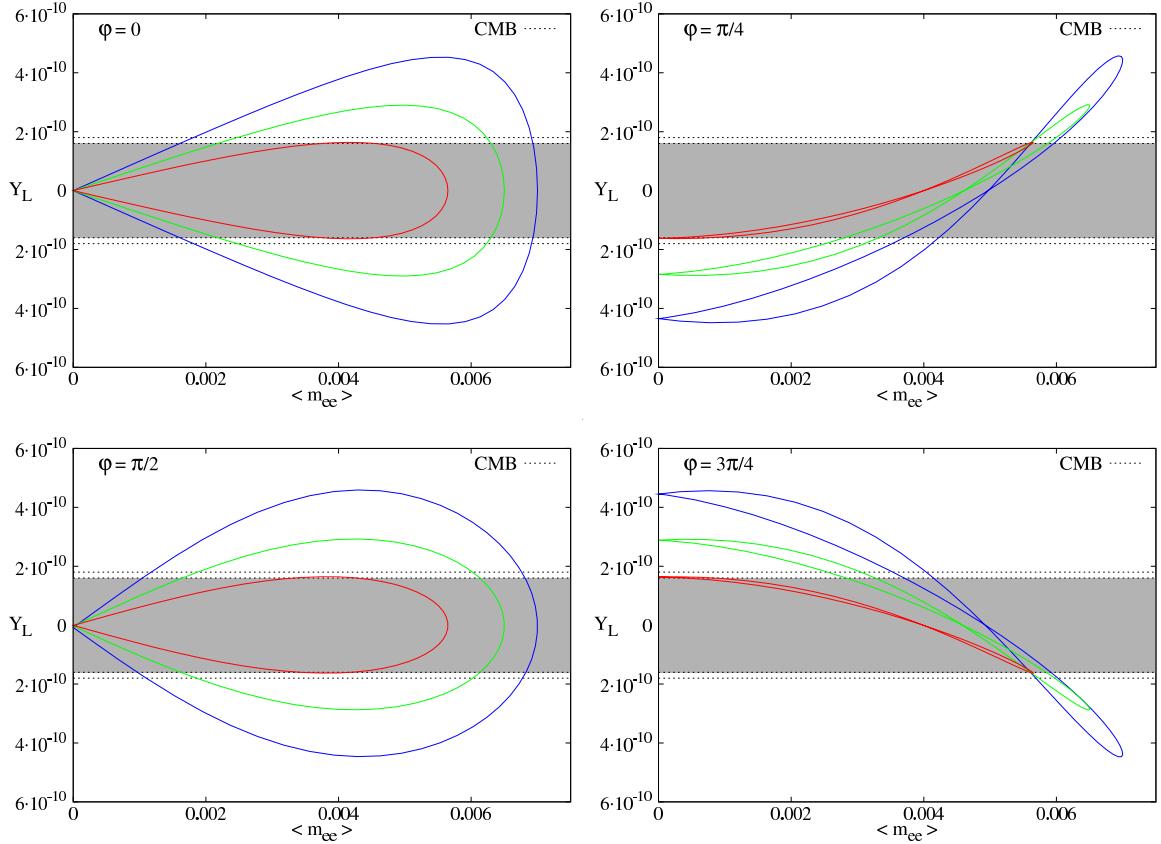


FIG. 5: (color online)  $Y_L$  vs.  $\langle m_{ee} \rangle$  for chosen values of Majorana phase. The large(blue), medium(green), and small(red) curves correspond to  $\sin^2 2\theta_{13} = 0.0255, 0.0164$ , and  $0.00922$ , respectively.

### Acknowledgments

This research was supported by the Chung-Ang University Research Grants in 2004.

---

[1] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81**, 1562 (1998)  
 S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **86**, 5651 (2001)

[2] Q. Ahmad *et al.* [SNO Collab.], Phys. Rev. Lett. **87**, 071301 (2001); Phys. Rev. Lett. **89**, 011301 (2002);

[3] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003)

[4] M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B **420** (1998) 397.

[5] M. V. Diwan *et al.*, Phys. Rev. D **68**, 012002 (2003) M. Aoki, K. Hagiwara and N. Okamura,

Phys. Lett. B **554**, 121 (2003) K. Hagiwara, Nucl. Phys. Proc. Suppl. **137**, 84 (2004)

[6] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[7] W. l. Guo and Z. z. Xing, Phys. Rev. D **67**, 053002 (2003) M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Lett. B **562**, 273 (2003) M. C. Gonzalez-Garcia and C. Pena-Garay, Phys. Rev. D **68**, 093003 (2003) M. Maltoni, T. Schwetz, M.A.Tortola and J. W. F. Valle, Phys. Rev. D**68**, 113010 (2003).

[8] P.F.Harrison, D.H.Perkins, W.G.Scott, Phys. Lett. B **458** (1999) 79; Phys. Lett. B **530** (2002) 167; Z.-z. Xing, Phys. Lett. B **533** (2002) 85; C. S. Lam, Phys. Lett. B **507** (2002) 214; X. He and A. Zee, Phys. Lett. B **560** (2003) 87; Phys. Rev. D **68** (2003) 037302 ; Y. Koide, Phys. Lett. B **574** (2003) 82;

[9] S. h. Chang, S. K. Kang and K. Siyeon, Phys. Lett. B **597**, 78 (2004)

[10] K. Siyeon, Phys. Rev. D **71**, 036005 (2005)

[11] S. Eidelman *et al.* [Particle Data Group], Phys. Lett. B **592**, 1 (2004).

[12] M. Gell-Mann, P. Ramond and R. Slansky, Print-80-0576 (CERN). T. Yanagida, Proceedings Of The Workshop On The Unified Theory And The Baryon Number In The Universe, R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980)

[13] V. Barger, D. A. Dicus, H. J. He and T. j. Li, Phys. Lett. B **583**, 173 (2004)

[14] E. W. Kolb and S. Wolfram, Nucl. Phys. B **172**, 224 (1980) [Erratum-ibid. B **195**, 542 (1982)].

[15] M. A. Luty, Phys. Rev. D **45**, 455 (1992). Phys. Lett. B **345**, 248 (1995) [Erratum-ibid. B **382**, 447 (1996)] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384**, 169 (1996) W. Buchmuller and M. Plumacher, Phys. Lett. B **431**, 354 (1998) W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A **15**, 5047 (2000)

[16] T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi and M. Tanimoto, Phys. Rev. Lett. **89**, 231601 (2002) S. Davidson and A. Ibarra, Nucl. Phys. B **648**, 345 (2003) T. Hambye, G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy, Phys. Rev. D **67**, 073025 (2003) A. de Gouvea, B. Kayser and R. N. Mohapatra, Phys. Rev. D **67**, 053004 (2003) S. Pascoli, S. T. Petcov and W. Rodejohann, Phys. Rev. D **68**, 093007 (2003) W. Grimus and L. Lavoura, J. Phys. G **30**, 1073 (2004) A. Ibarra and G. G. Ross, Phys. Lett. B **591**, 285 (2004) S. Davidson and R. Kitano, JHEP **0403**, 020 (2004) M. C. Chen and K. T. Mahanthappa, Phys. Rev. D **71**, 035001 (2005)

[17] C. L. Bennett *et al.* Astrophys. J. Suppl. **148**, (2003)1 D. N. Spergel *et al.* Astrophys. J.

Suppl. **148**, (2003)175

- [18] M. Fukugita and T. Yanagida, Phys. Lett. **B 174**, 45 (1986).
- [19] J. A. Harvey and M. S. Turner, Phys. Rev. D **42**, 3344 (1990). E. W. Kolb and M. S. Turner, "The Early Universe," Addison-Wesley, 1990 H. B. Nielsen and Y. Takanishi, Phys. Lett. B **507**, 241 (2001)